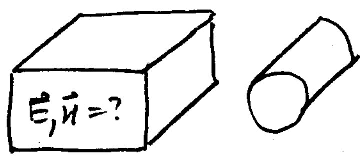
EM Solution Methods

There are two basic types of EM problems we can solve analytically:

① Canonical boundary geometry



Modal Solutions

1. Homogeneous solutions (modes of free space)
2. Apply boundary condition $\Rightarrow$ Modes
3. Forcing function (current source) $\Rightarrow$ Unique linear combination of modes

② Current in free space

BC = radiation condition (usually)

$\vec{E}, \vec{H} = ?$

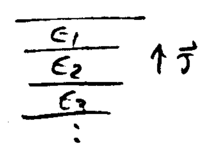
Green's function method

Integrate to find $\vec{E}, \vec{H}$ using radiation integral
$$\vec{E} = \bar{\bar{G}} * \vec{J}$$

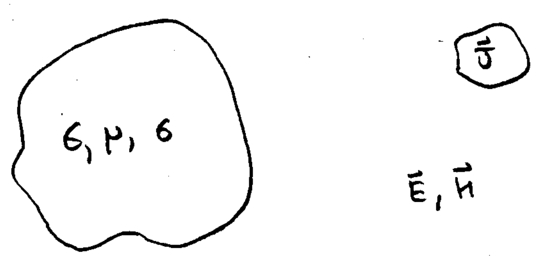
Radiation B.C. is most common $\Rightarrow$ free space Green's function

(other B.C.'s or materials if Green's function is available ... construct using modal method, etc.)

Example: Layered media



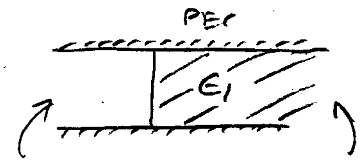
With these two classes of methods, we can only solve a limited number of problems. In general, we have sources and complex material structures:



General problems of this type must be handled using more advanced methods, including numerical methods.

Types of methods:

1. Mode matching: Numerical extension of modal method to multiple regions:



$\vec{E} = \sum a_n \vec{E}_n^{(1)}$ $\qquad$ $\vec{E} = \sum b_n \vec{E}_n^{(2)}$

Match at interface $\Rightarrow$ set of linear equations for a_n, b_n

2. Difference methods: approximate Maxwell's equations

Finite Difference (FD) – Laplace, Poisson problems, Time-harmonic fields

Finite Difference Time Domain (FDTD)

Finite Element Method (FEM) – general grids (non-rectangular)
– variational principle $\rightarrow$ Rayleigh-Ritz

3. Equivalent current sources – transform Maxwell's equation into integral equations – EFIE, MFIE, CFIE

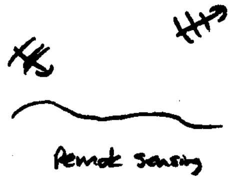
Method of Moments (MOM)

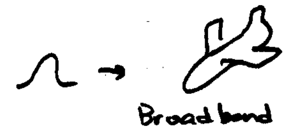
– surface integral equations – PEC, homogeneous dielectrics
– bodies of revolution (axisymmetric)
– volume – inhomogeneous materials
– Time-domain – recent research – (stability is hard)

4. FEM/BEM – FEM on volume; BEM on outer boundary as absorbing boundary condition (MOM)

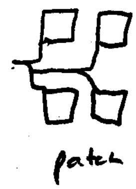
	Advantages	Disadvantages
FD	Easy to implement	Exterior problems require ABC
FDTD	Easy to add circuit elements (nonlinear) since time-domain Sparse matrix	Dispersion limits accuracy Stair-stepping of mesh
MOM -surface	Surface unknowns only required Accurate -no dispersion error	Limited to PEC/homogeneous problems Dense matrix (FMM...) Frequency domain -single frequency
-volume	Fast using CG-FFT	requires volume mesh
FEM	Flexible - geometry, materials Sparse matrix	Requires ABC

RCS/scattering

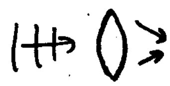
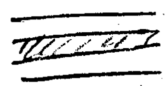





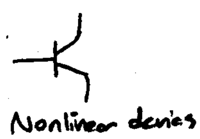
Antennas

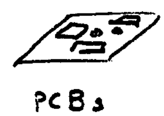


OPTICS

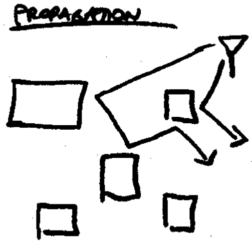
CIRCUITS



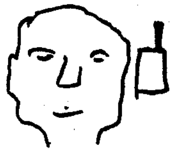


EMC/EMI

PROPAGATION



BIOMEDICAL:



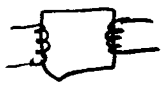
WAVEGUIDES

modes

bends

Microstrip

MAGNETICS



Losses, eddy currents

FD/FDTD

1D - Derive stencils
Stability
Dispersion

2D - Stencil
Eigenvalue / waveguide problems - FD
Dielectric interface
Dirichlet, Neumann BC's

FDTD - Yee cell - 2D TM/TE, 3D
Sources - hard, soft
Materials
Grid truncation - Mur, PML (ABCs)
Scattered field formulation
Post-processing
- Far field - radiation integral
- RCS, SW, SA
Stability

Numerical Integration

Polynomial rules - error (fixed nodes)
Extended rules
Gaussian quadrature (free weights/nodes)
Weight function - singularities

MOM

Equivalent currents, integral equations
2D - TM/TE - EFIE - surface - PEC
Weighted residuals - MOM, basis functions, testing functions
Far field
3D MOM
Computational cost
Error analysis

Volume MOM

Linear systems

Direct solution, LU
Iterative solvers
Stationary - Jacobi, Gauss-Seidel
Krylov subspace - convergence rates, condition number
Preconditioners

FEM

Variational principles

Functionals

Rayleigh-Ritz

Elements, shape functions (basis)

Assembly

Examples:

Laplace – free, fixed nodes

Helmholtz – eigenvalue probs, scattering

Boundary conds. – Dirichlet, Neumann

– ABC

h, p – refinement, edge elements (3D)

FEM – BEM

Inverse Problems

Ill-posed problems, regularization

Inverse scattering

Regularized sampling